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T1 ________________	65122	F1 ________________
T2 ________________	Problem Chosen	F2 ________________
T3 ________________	D	F3 ________________
T4 ________________		F4 ________________

2017
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Summary Sheet

How to estimate the amount of time security check needs has long been a headache for both airport staffs and passengers. In order to develop an optimized security check system and narrow the variance of the check time for passengers, this paper deploys a Queuing Theory model to analyze the current security check process.

First, we regard the current security check process as a multi-single server system and define an index of average stay time to measure the efficiency of each zone. The zone with longest average stay time W_q is the bottleneck which need to be improved.

The fitting result shows that both the arrival intervals and the service intervals are under the negative exponent distribution. After calculating the parameter λ of the arrival interval curve and the parameter μ of the service interval curve, we find that Zone B is the area with longest average stay time. And in the zone B, millimeter scanner is the bottleneck with lowest work capacity.

Then, based on the work efficiency gap of each zone, we propose three practical schemes to accelerate the check speed, which are adjustment on proportion between precheck and regular lanes, change on server model (merge several queues into one), increasing the facility of millimeter wave scanner. Through these three methods, we balance the use rate of different lanes, shorten the waiting time and eliminate the bottleneck part.

Next, by applying the Theory of Grey Box System, we focus on the impact the passenger background brought to the check speed. We use the method of Correlation Analysis to figure out the relationship between four variables and the security check speed. The four variables are Age, Income, Gender and Flying miles. This analysis process serves as the sensitivity test of out model, and in the future, we can more variables based on personal characteristics or cultural difference to make further improvement of the model.

In the sensitivity analysis, we find that Gender and Income have more impact on the check time than Age and Flying miles. Because Gender has much influence and we are inspired by a practice of a small airport, we try to optimize our model by offering separate female privileged lanes. And the result is that the average stay time has shortened by 65%.

Though the effect of our model is quite good, our model is based on many simplified assumptions. So in the policy and procedure suggestion given to the security manager, we not only conclude the results the practical improve methods got from our model, but also talk about the efficiency of the airport staffs, the time spent on linking process, etc.

We see our model as a successful one for it has stable math support, but in further improvement, we may consider to delete more simplified assumption to make the model more adjustable to reality. The further research can include: changing the arrival parameter more timely to simulate the real crowd flow in an airport, more thorough inspection of the mechanism behind the impact cultural variables bring to the security check process.

Keyword: The Queuing Theory; The Theory of Grey Box System

An Optimized Airport Checkpoint Model Based on Queuing Theory

Control Number #65122

Janauary,2017

Contents

1 Introduction

1.1 Background

All airports want to fulfill the goal of ensuring security against any terrorist attack and bringing satisfactory experience to passengers, but to promise both is not easy. The conflict between security and convenience must be solved. Passengers need a shorter process of security check as well as no decrease in the safety index. Though some airports have make efforts on adding more staffs to cut the queue length, this is not the only effective way. Considering the high human resource cost and the low use rate of new lanes besides rush hours, an optimized queue model with deploying strategies is a better choice.

1.2 Restatement of the Problem

We are required to establish a model to explore the average stay time on each zone of a passenger through a security check process. The zone with longest stay time is the bottleneck zone.

Then we develop some modification models by changing the security check process or providing classified service to different demographic groups. The modifications realize increasing the number of passengers serviced every time unit and decreasing the variance of time the whole check process use.

The optimized model considers the personal characters including cultural difference, behavior customs etc. to further undermine the variance of check time. Finally, we simplify the technical details and offer applicative advice to managers to make the model user-friendly in specific environment.

2 Assumptions and Justifications

- **Passenger arrivals come from an infinite or very large population.** Because we are required to figure out the jam problem during rush hour, we assume the passenger number is very huge, neglecting the off-season period.
- **The arrival way of passengers is independent (one by one).** Not considering big travel groups, most passengers choose the time to arrival on their own or with a very small group, so we regard the arrival of passengers as a random incident. The only parameter is the flight time. In further models, we may consider adding the condition of big travel groups to make optimization.
- **The arrival time of passengers is the same as the time they enter the security check queue.** Most passenger want to secure their punctuality for not missing the flight, so they tend to join the queue as soon as possible.
- **The average service rate is faster than the average arrival rate.** Otherwise, the queue will tend to infinity. It is not the same the situation in reality.
- **The work efficiency has no difference among several parallel service windows in the same check zone.** The staffs are well-trained with working skills, so the subtle difference between staffs can be neglected. We consider the various personal condition of passengers as the main factor causing impact to check speed, such as luggage volume and behavioral customs.
- **The Passengers are treated on a FIFO basis with no behavior of cutting in the queue.**

3 Notation

Name	Definition	Denotation
The average length of the passenger line	The mathematical expectation of passenger number in i zone system (including the passenger in service and the passengers waiting for this zone)	L_{si}
The average length of the queue	The mathematical expectation of passenger number in the waiting queue of the zone i	L_{qi}
The average stay time	The mathematical expectation of time a passenger spent in the whole process of the zone i (including the waiting time in the queue and the time in service)	W_{si}
The average waiting time	The mathematical expectation of time a passenger spent in the queue of zone i	W_{qi}
The arrival parameter	The parameter of the fitting negative exponent distribution about arrival time in zone i	λ_i
The service parameter	The parameter of the fitting negative exponent distribution about service time in zone i	μ_i

4 Analysis of the Current Process

4.1 The Basic Model

- **The arrival time interval is mainly under the negative exponential distribution.** We use Matlab fitting tool to get this basic assumption. The charts and values are in the following analysis section.
- **The service time of each zone is mainly under the negative exponential distribution.** We use Matlab fitting tool to get this basic assumption. The charts and values are in the following analysis section.
- **The airport security model is a multi-single server model**, for there is a separate queue lined up before every check server. Though there are several parallel servers in one zone, we still call the system a single server model, or multi-single server model.
- **The whole security system is a waiting system (call-delay system).** When a passenger arrives at the check point, if all the servers are occupied, the passenger will join the queue and wait until they get the service before they leave.

Now we use the concept *Birth and Death process* to pre-analyze the security check service and get some basic formula we need to use later.

We use $N(t)$ to represent the number of passengers in the system at the time point t, clearly $\{N(t), t \geq 0\}$ is a random process. We call the security check random process a *Birth and Death process* when it meets the following three conditions (Shou kui, Si,2011): [1]

1. If $N(t) = n$ then the time periods from the time point t to the time point the next passenger arrive are under the negative exponent distribution with a parameter λ_n , $n = 0,1,2,\cdots$.

2. If $N(t) = n$ then the time periods from the point t to the time point the next passenger leave are under the negative exponent distribution with a parameter μ_n , $n = 0,1,2,\cdots$.

3. Only one passenger arrives or leaves at a time point.

Supplement: We divide a day into m time periods, each time period has its own λ_n . With access to the TSA statistics, we can update the value of λ every one hour, so we divide each day into 24 periods. And there are three methods that we can get λ , the first one is the statistics from checkpoints, the second one is the timely detection at the entrance of the airport, the last one is prediction from the flight timetable.

Now we specifically explain the last method in detail. We assume there are totally M flights scheduled to take off in a certain day and we divide the day into m time periods, then there are $\overline{M} = \frac{M}{m}$ flights to take off in each time period on average. We choose the time period with the flight number closest to $\overline{M}$, mark it as time period k , and its flight number as M_k .We use M_n to represent the flight number of other period, then based on λ_k ,we can get the value of λ_n from the formula $\lambda_n = \frac{\lambda_k M_n}{M_k}$.

Then we need to get the state distribution when the system reach the equilibrium state. It's easy to notice the average occurrence rate is proximately equal, for the movement of arrival and leaving happens in turn. Using this principle, we can get the following equilibrium state formula at any state:

$$\begin{array}{ll} 0 & \mu_1 p_1 = \lambda_0 p_0 \\ 1 & \lambda_0 p_0 + \mu_2 p_2 = (\lambda_1 + \mu_1) p_1 \\ 2 & \lambda_1 p_1 + \mu_3 p_3 = (\lambda_2 + \mu_2) p_2 \\ \vdots & \vdots \\ n & \lambda_{n-1} p_{n-1} + \mu_{n+1} p_{n+1} = (\lambda_n + \mu_n) p_n \\ \vdots & \vdots \end{array} \tag{1}$$

We can get the general recursion formula

$$n: \ p_{n+1} = \frac{\lambda_n}{\mu_{n+1}} p_n + \frac{1}{\mu_{n+1}} (\mu_n p_n - \lambda_{n-1} p_{n-1}) = \frac{\lambda_n}{\mu_{n+1}} p_n = \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \mu_n \cdots \mu_1} p_0$$

We use set up a new notation

$$C_n = \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \mu_n \cdots \mu_1}, n = 1, 2, \cdots \tag{2}$$

then the distribution of equilibrium state is

$$p_n = C_n p_0, n = 1, 2, \cdots \tag{3}$$

Using the equation $\sum_{n=0}^{\infty} p_n = 1$, we have

$$p_0 = \frac{1}{1 + \sum_{n=1}^{\infty} C_n} \tag{4}$$

Having got all the preparation, now we start out queue model. In the following analysis, we use M to refer to exponential distribution, and 1 refer to single server model, so we abbreviate the current security check process as c * M/M/1 model. It means a multi-single server system, with both passenger arrival time interval and service time under negative exponential distribution.

We use $p_n = P\{N = n\}(n = 0,1,2\cdots)$ to represent the probability distribution of queue length N after the system reaches equilibrium state. Then form equation (2)~(4), we notice that $\lambda_n = \lambda, n = 0,1,2,\cdots$ and $\mu_n = \mu, n = 0,1,2,\cdots$ We define $\rho = \frac{\lambda}{\mu}$, assuming that $\rho < 1$, otherwise the queue length will extend to infinity. Then

$$C_n = \left(\frac{\lambda}{\mu}\right)^n, n = 1,2,\cdots$$

So

$$p_n = \rho^n p_0, n = 1,2,\cdots$$

And the p_0 here means

$$p_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \rho^n} = \left(\sum_{n=0}^{\infty} \rho^n\right)^{-1} = \left(\frac{1}{1-\rho}\right)^{-1} = 1 - \rho \tag{5}$$

Hence,

$$p_n = (1-\rho)\rho^n, n = 1,2,\cdots \tag{6}$$

We get the probability of n passengers remaining in the system when it reaches equilibrium state from formula (4) and (5) and ρ represents the probability of the server being occupied. Only when $\rho = \frac{\lambda}{\mu} < 1$,which means the average arriving rate of passenger is less than the average service rate of the system, can the whole system reach the equilibrium state.

Now we can get the formula of the variable $L_{si}, L_{qi}, W_{si}, W_{qi}$ we set before.(All the variable here is about the M/M/1 model, not the c*M/M/1 model)

The average length of the passenger line

$$
\begin{aligned}
L_s &= \sum_{n=0}^{\infty} np_n \\
&= \sum_{n=1}^{\infty} n(1-\rho)\rho^n \\
&= (\rho+2\rho^2+3\rho^3+\cdots)-(\rho^2+2\rho^3+3\rho^4+\cdots) \\
&= \rho+\rho^2+\rho^3+\cdots \\
&= \frac{\rho}{1-\rho} \\
&= \frac{\lambda}{\mu-\lambda}
\end{aligned} \tag{7}
$$

The average length of the queue

$$L_q = \sum_{n=1}^{\infty}(n-1)p_n = L-(1-p_0) = L-\rho = \frac{\lambda^2}{\mu(\mu-\lambda)} \tag{8}$$

The waiting and service time are both under the exponent distribution, so the stay time is under the negative exponent distribution with parameter $\mu-\lambda$. That is to say $P\{T>t\}=e^{-(\mu-\lambda)t}, t\geq 0$.

Hence, the average stay time

$$W_s = \frac{1}{\mu-\lambda} \tag{9}$$

Because stay time T consists of waiting time T_q and service time V,

$$W_s = E(T) = E(T_q)+\mathrm{E}(V) = W_q + \frac{1}{\mu} \tag{10}$$

The average waiting time

$$W_q = W_s - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} \tag{11}$$

4.2 Model Application

4.2.1 Data Cleaning and Explanation

The explanation of our use of the data

We use *TSA Pre-check Arrival Times(hour:min:sec)* and *Regular Pax Arrival Times(hour:min:sec)* to calculate arrival interval. The *ID Check Process Time(sec)* is the service time of zone A. The *Millimeter Wave Scan Times(sec)* represents the time of body scanning in zone B, while the *X-ray Scan Time(sec)* represents the time of luggage scanning in zone B. The *time to get scanned property(min)* is the service time of the whole zone B.

The definition of bottleneck

The most concerning variable of passengers is the time expenditure of check process, so the bottleneck zone of the whole process must be the one with longest stay time. The stay time consists of two parts, the waiting time and the checking time. To locate the exact bottleneck of current process, we only need to compute the W_s of each zone.

4.2.2 Zone Analysis

① Zone A

Using the method above, we get the arrival data below:

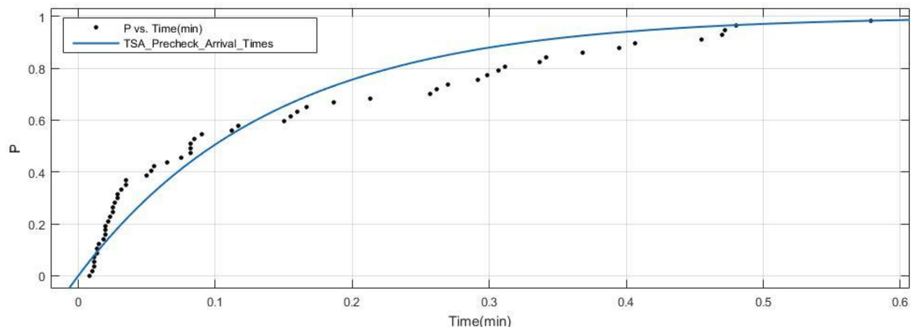


Figure 1: the fitting distribution of precheck arrival intervals

The arrival parameter of precheck lane $\lambda_p = 7.055$

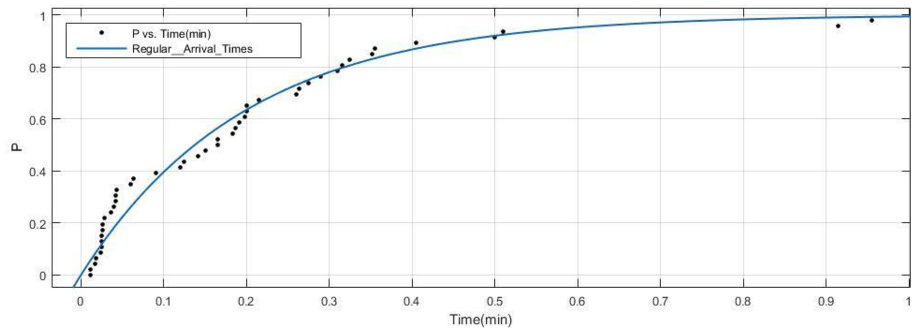


Figure 2: the fitting distribution of regular arrival intervals

The arrival parameter of regular lane $\lambda_r = 5.04$

Using the method above, we get the service data below:

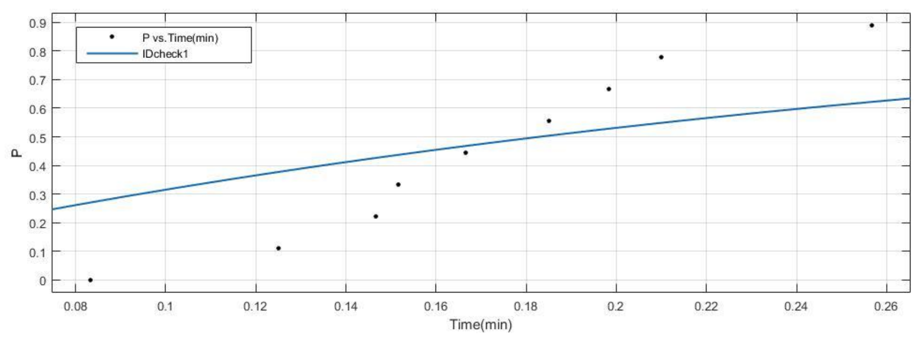


Figure 3: the fitting distribution of service time in zone A

The service parameter of zone A server 1 $\mu_{a1} = 3.769$

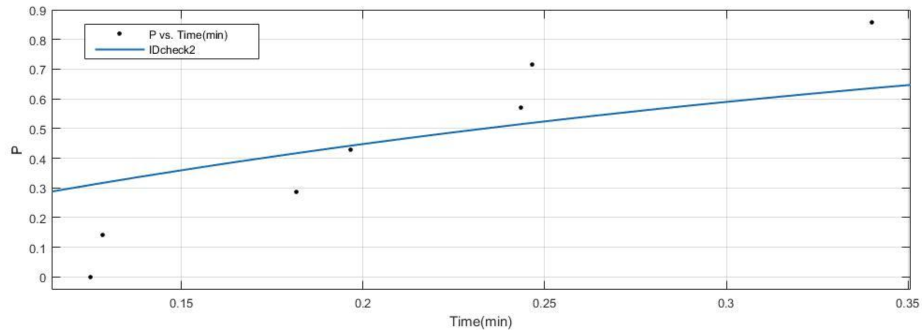


Figure 4: the fitting distribution of service time in zone A

The service parameter of zone A server 2 $\mu_{a2} = 2.968$

The ID check process has no difference in zone A between the precheck lane and the regular lane, so we use the mean value of server 1 and server 2 $\mu_a = (\mu_{a1} + \mu_{a2})/2 = 3.3685$ to represent the service parameter of zone A.

Because of the limit of airport scale, the queue length cannot tend to infinity, so the **we assume that the service capacity is larger than the passenger arrival volume**, the problem is only the low speed, so the server number n must meet the inequation $\frac{\lambda}{n} < \mu$

Hence, the number of precheck lanes in zone A $n_{pa} > \frac{\lambda_p}{\mu_a} = \frac{7.055}{3.3685} = 2.094$

To simplify the analysis of the current process, we decide to use the smallest server number meeting this standard, so the number of precheck lanes in zone A is $n_{pa} = 3$, and according to the 1:3 data, the number of regular lanes in zone A is $n_{ra} = 9$

② zone B

Because the passenger arriving at zone A earlier does not necessarily arrive at zone B earlier (the passenger sequence is changing during the service process due to the subtle difference on speed of different servers), we decide to simplify the arrival model of zone B. **We regard the arrival time at zone A plus the average stay time of zone A the same as the arrival time at zone B.** According to this assumption, the arrival parameter of zone B equals to the arrival parameter of zone A, so when we use the arrival parameter, we do not distinguish zone A and zone B, we only use λ_p and λ_r to distinguish precheck lanes and regular lanes.

In zone B, we first use *the time to get scanned property* to calculate the service parameter μ_{rb}, seeing zone B as an integration.

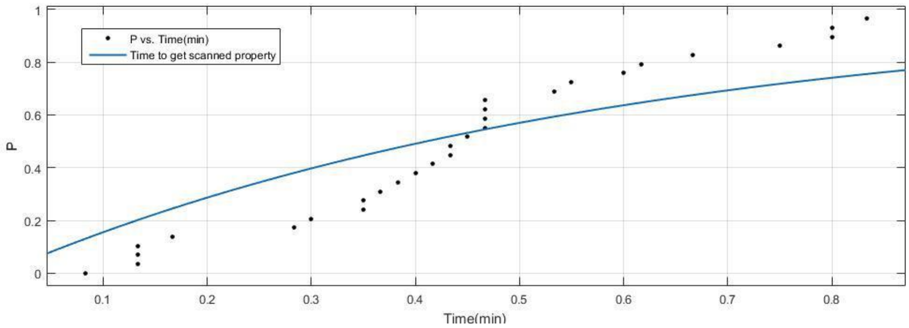


Figure 5: the fitting distribution of service time in zone B

The service parameter of regular lanes in zone B μ_{rb} = 1.689

No data provided to distinguish the speed difference in zone B between precheck and regular lanes, but after searching for the statistics of several large-scale airports, **we assume that the service capacity of precheck lanes is 50% higher than the regular lanes(US Census Bureau)**.[9] Hence,

The service parameter of precheck lanes in zone B μ_{pb} = 2.5335

Also, to simplify the analysis, we choose the smallest proper number of lanes for zone B. When the precheck lane number is 2, the average waiting time W_{qpb} would be over 100 minutes, but when the number is 3, W_{qpb} is ideal. Our model wants to approach the proper server number as well as locate the bottleneck zone, so we choose the least number of precheck lanes which obeys the inequation $n_{pb} \geq \frac{\lambda_{pb}}{\mu_{pb}}$ （in order to meet the assumption that the service capacity is larger than the arrival rate） in zone B $n_{pb} = 4$. According to the proportion of 1:3, the number of regular lanes in zone B is $n_{rb} = 12$. Then we calculate the value of variables as below:

Value of variables	Precheck lanes	Regular lanes
Zone A	Lane number 3 $L_{sa} = 2.3127$ $L_{qa} = 1.6146$ W_{sa} = 0.9834 W_{qa} = 0.6866	Lane number 9 $L_{sa} = 0.1994$ $L_{qa} = 0.0331$ W_{sa} = 0.3561 W_{qa} = 0.0592
Zone B	Lane number 4 $L_{sb} = 2.29133$ $L_{qb} = 1.59516$ W_{sb} = 1.29912 W_{qb} = 0.904412	Lane number 12 $L_{sb} = 0.330969$ $L_{qb} = 0.0823014$ W_{sb} = 0.788022 W_{qb} = 0.195956

Because W_{qb} is much longer than W_{sa} , which means the average stay time of passengers in longer in zone B than in zone A, so the passengers who finished the check in zone A may be stuck in zone B, for the people before haven't finished check yet. Hence, we conclude that zone B is the bottleneck zone. Now we subdivide zone B to see the exact bottleneck step.

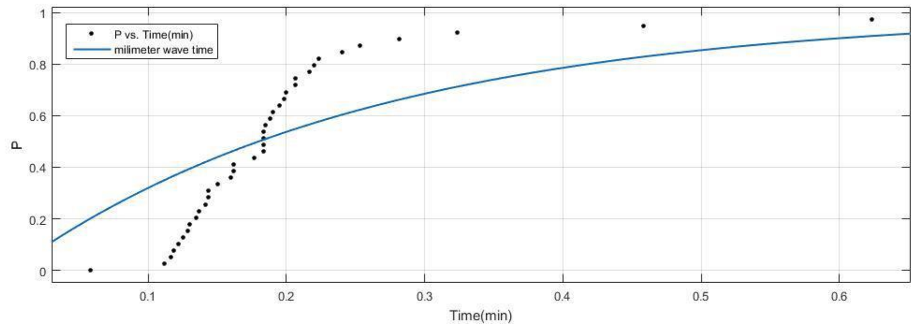


Figure 6: the fitting distribution of millimeter wave scanner service time

The service parameter of millimeter wave scanner $\mu_m = 3.856$

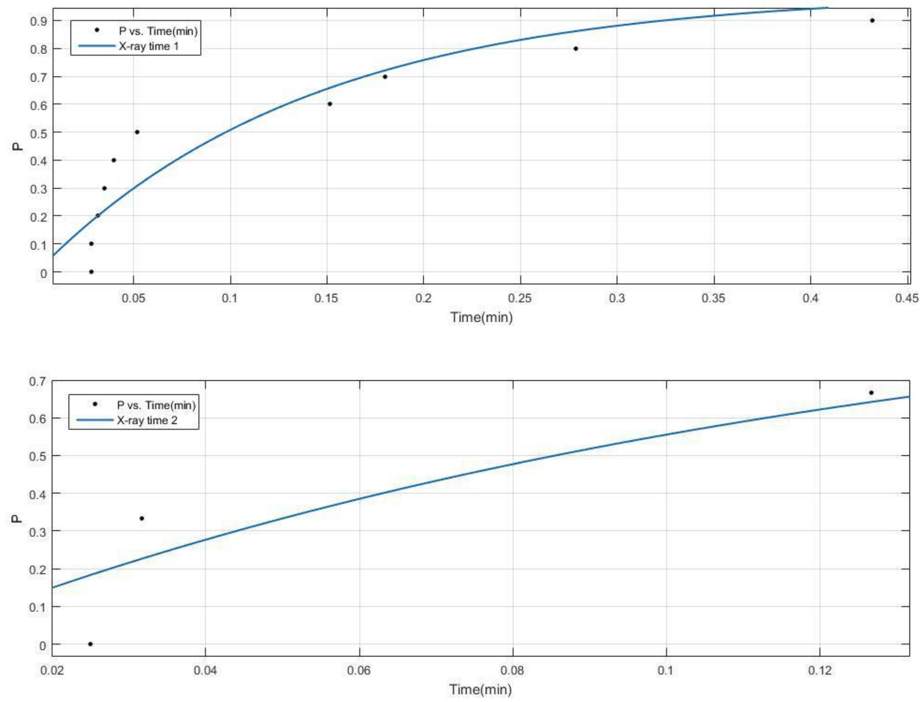


Figure 7: the fitting distribution of X-ray scanner service time

We use the mean value of the two servers to represent the service parameter of X-ray scanner, and we get $\mu_x = 7.6$.

Because $\mu_m < \mu_x$,the service capacity of x-ray is much higher than the millimeter capacity, we confirm that millimeter scanner is the exact bottleneck of zone B.

Conclusion

The bottleneck of the whole process is zone B, and in which the body scan process is the key bottle neck, for it cost the most W_s.

5 Modification Model

5.1 Modification on Lane Proportion

The reason why precheck program cannot get the ideal effect is because too many passengers are using precheck lanes. And the stuck zone is zone B, for the 1:3 proportion doesn't adapt to the proportion of passengers using precheck and passengers who not, so we need to changer the proportion of lanes offer to precheck and regular passengers. The model we set up has confirmed that the proper precheck lane number in zone B is 4, under this situation, 12 regular lanes are not fully occupied. After calculation, we find cutting the regular lane in zone B to 4 can still meets the need. After modification, new value of all the measure variables are as below:

Value of variables	Precheck lanes	Regular lanes
Zone A	Lane number 3 $L_{sa} = 2.3127$ $L_{qa} = 1.6146$ $W_{sa} = 0.9834$ $W_{qa} = 0.6866$	Lane number 2 $L_{sa} = 2.96995$ $L_{qa} = 2.22184$ $W_{sa} = 1.17855$ $W_{qa} = 0.881682$
Zone B	Lane number 4 $L_{sb} = 2.29133$ $L_{qb} = 1.59516$ $W_{sb} = 1.29912$ $W_{qb} = 0.904412$	Lane number 4 $L_{sb} = 2.93706$ $L_{qb} = 2.19106$ $W_{sb} = 2.331$ $W_{qb} = 1.73894$

5.2 Modification on Server Model

With the support of the math calculation, we decide to transfer the c*M/M/1 model to M/M/c model (Here we use variable c to represent the number of servers). That means the several separate queue lines before every lane will be merged into one queue line, no matter which lane become spare, the first passenger of the only queue come to get service (Dian yuan, Meng,2011). [2]

Because there are c servers, the variable μ_n we set up before changes while λ_n remains unchanged.

$$\mu_n = \begin{cases} n\mu, n = 1, 2, \cdots, c \\ c\mu, n = c, c+1, \cdots \end{cases}$$

Formula (5) also changes

$$C_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!}, n = 1, 2, \cdots, c \\ \dfrac{(\lambda/\mu)^c}{c!}\left(\dfrac{\lambda}{c\mu}\right)^n = \dfrac{(\lambda/\mu)^n}{c!c^{n-c}}, n \geq c \end{cases}$$

$$p_n = \begin{cases} \dfrac{\rho^n}{n!} p_0, n = 1, 2, \cdots, c \\ \dfrac{\rho^n}{c!c^{n-c}} p_0, n \geq c \end{cases}$$

The p_0 here means

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!(1-\rho_c)} \right]^{-1} \tag{12}$$

Then we can get the formula to calculate the value of variables we set up as below:

$$L_q = \sum_{n=c+1}^{\infty} (n-c) p_n = \frac{p_0 \rho^c}{c!} \sum_{n=c}^{\infty} (n-c) \rho_c^{\ n-c} = \frac{p_0 \rho^c}{c!} \frac{d}{d\rho_c} \left(\sum_{n=1}^{\infty} \rho_c^{\ n} \right) = \frac{p_0 \rho^c \rho_c}{c!(1-\rho_c)^2} \tag{13}$$

$$L_s = L_q + \rho \tag{14}$$

$$W_s = \frac{L_s}{\lambda}, W_q = \frac{L_q}{\lambda} = W_s - \frac{1}{\mu} \tag{15}$$

After modification, both the average length of passenger line and queue have no change, but the average stay time and waiting time have been shortened to a large scale.

Value of variables	Precheck lanes	Regular lanes
Zone A	Lane number 3 $L_{sa} = 2.3127$ $L_{qa} = 1.6146$ $W_{sa} = 0.327815$ $W_{qa} = 0.228859$	Lane number 2 $L_{sa} = 2.96995$ $L_{qa} = 2.22184$ $W_{sa} = 0.589275$ $W_{qa} = 0.440841$
Zone B	Lane number 4 $L_{sb} = 2.29133$ $L_{qb} = 1.59516$ $W_{sb} = 0.324781$ $W_{qb} = 0.226103$	Lane number 4 $L_{sb} = 2.93706$ $L_{qb} = 2.19106$ $W_{sb} = 0.582751$ $W_{qb} = 0.434734$

5.3 Modification on millimeter wave scanners

The bottleneck part inside zone B is the millimeter wave scanners, for the service capacity μ of the X-ray is much higher than the millimeter wave. When the luggage scanning has finished, passengers are still waiting to get body scanned, so we decide to increase the facility number of this bottleneck.

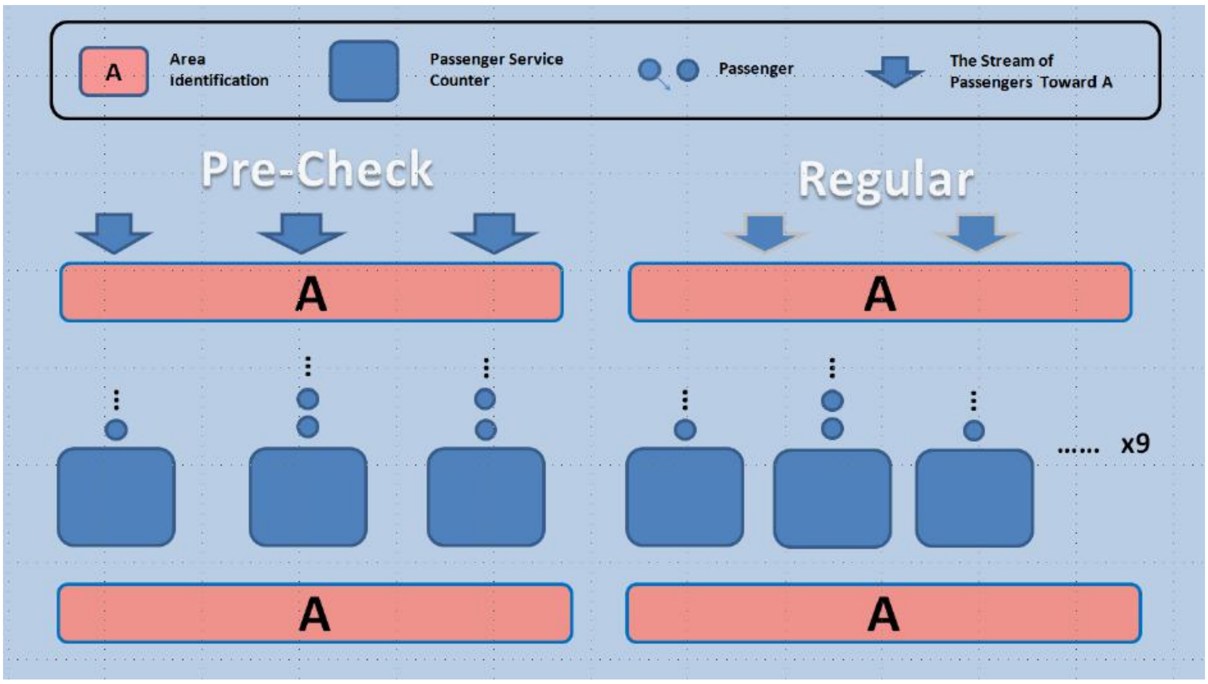
To keep the service speed at a similar level with the arrival speed in zone B, **we recommend to double the facility of millimeter wave scanners**, after calculation.

After modification, the service parameter of millimeter wave scanners change from $\mu_m = 3.856$ to $\mu_m = 7.712$, while the service parameter of X-ray scanner remains $\mu_x = 7.6$. The speed gap is largely narrowed.

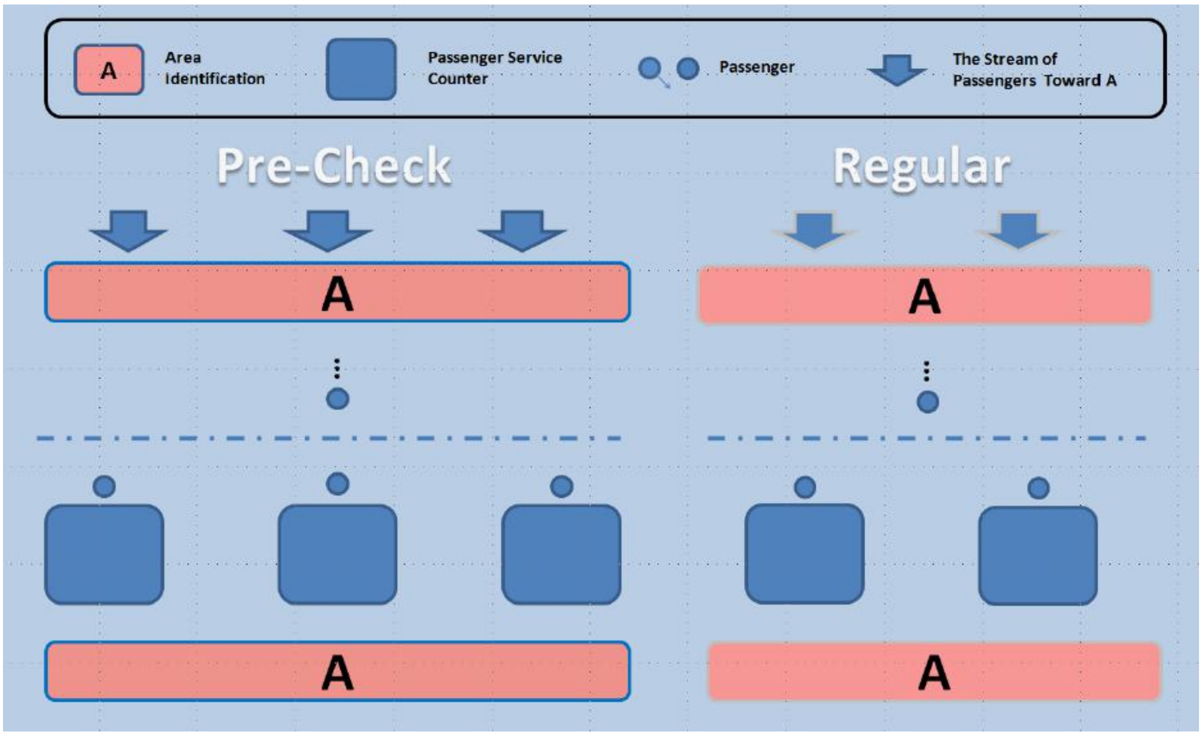
Value of variables	Before double	After double
The bottleneck of zone B	Millimeter wave scanner	X-ray scanner
Precheck zone B	$L_{sb} = 0.842992$ $L_{qb} = 0.385588$ $W_{sb} = 0.119489$ $W_{qb} = 0.0546546$	$L_{sb} = 0.302206$ $L_{qb} = 0.0701337$ $W_{sb} = 0.0428357$ $W_{qb} = 0.0099409$
Regular zone B	$L_{sb} = 0.485362$ $L_{qb} = 0.158599$ $W_{sb} = 0.096302$ $W_{qb} = 0.031468$	$L_{sb} = 0.198738$ $L_{qb} = 0.0329487$ $W_{sb} = 0.0394322$ $W_{qb} = 0.00653744$

The Effect Diagram

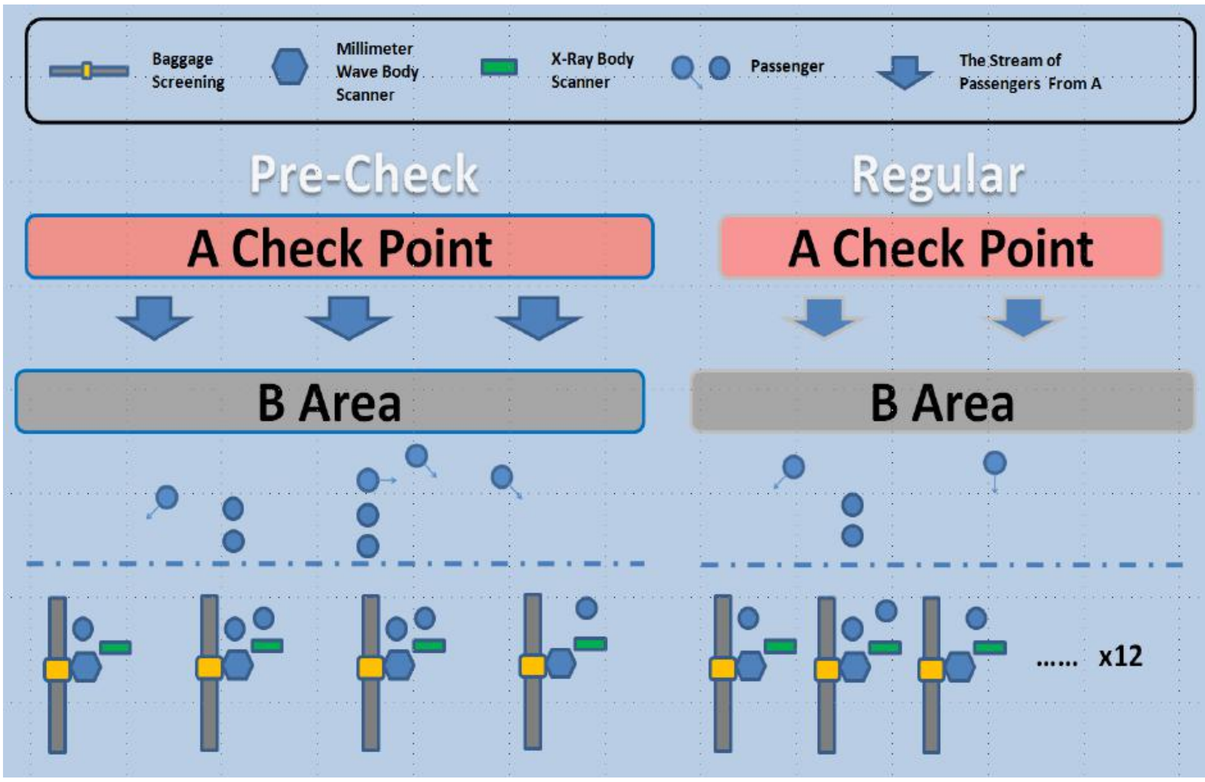
Zone A Before Modification



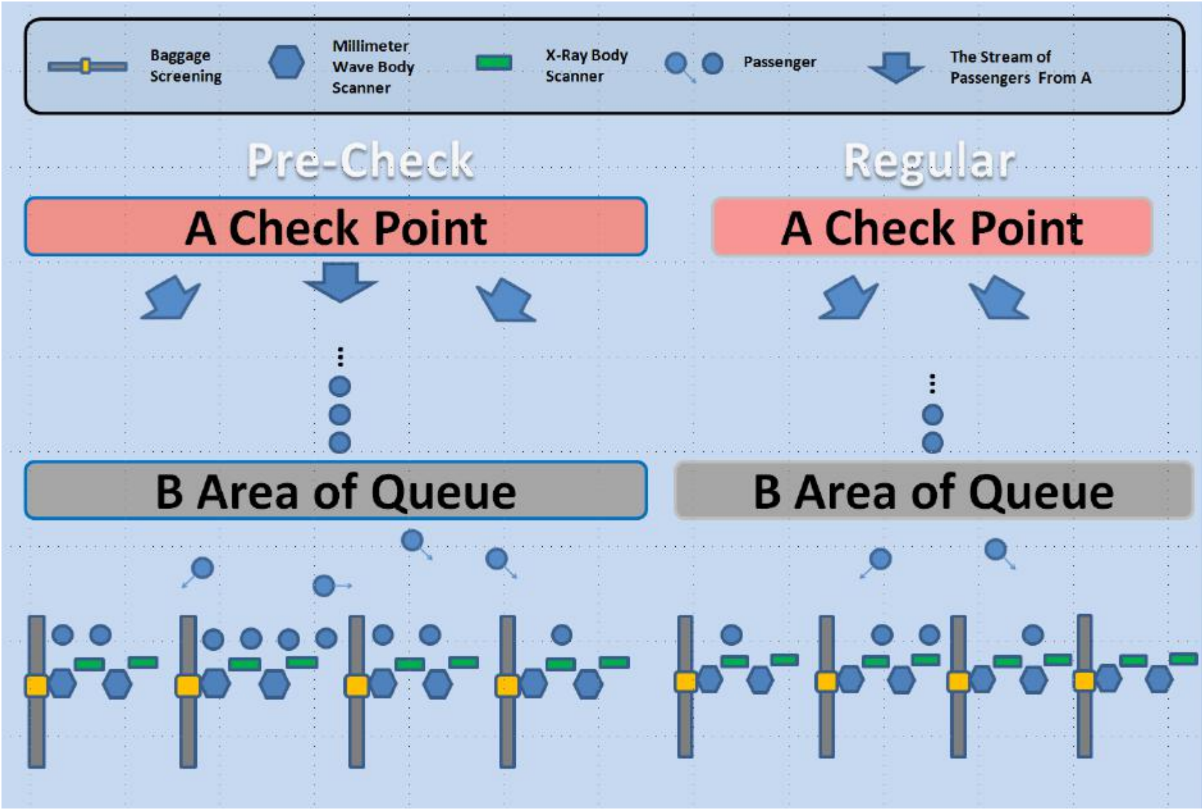
Zone A After Modification



Zone B Before Modification



Zone B After Modification



6 Further Model Based on Passenger Characteristics

6.1 Sensitivity Analysis

The modified model we propose has highly shortened the time spent on security check process, but it is just a model based on many simplified assumptions. In reality, different cultures have various styles which may break our assumptions. To promise the efficiency, we need to consider much more parameters and do quantitative research of the cultural factors (Jacques Calmet,2017). [3]

In searching for the data of cultural influence on security check speed, we find that **the relation between personal character is tighter than that between macroscopical culture and check speed.** That is to say, variables such as age and income level weigh more on determine airport strategy, for it is clear that the old behave slower but not clear if a Swiss would move faster than an American. Also, demographic statistics are easy to get and analyze.

We decide to use *2016 SFO Customer Survey Data* [4] to do sensitivity analysis, how our model may be influenced by the travelers' style.

The four variables we select to analyze are:

- Age
- Income
- Gender
- Flying miles per year

The proportion of variable value can be roughly shown as below:

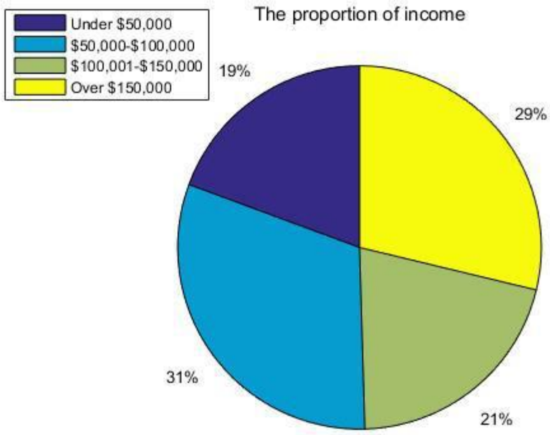


Figure 8: The income distribution of passengers in SFO (2016)

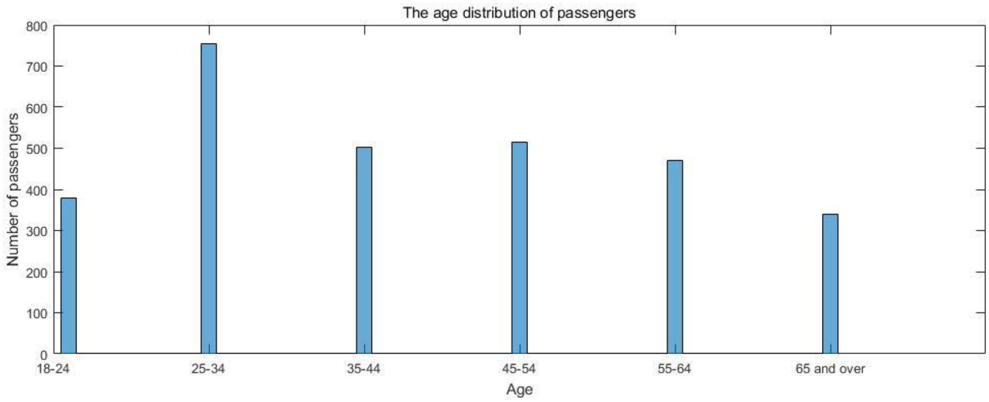


Figure 9: The age distribution of passengers in SFO (2016)

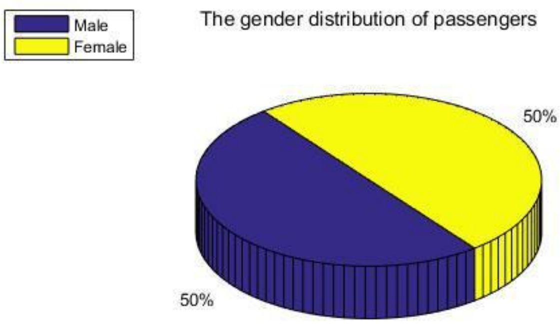


Figure 10: The gender distribution of passengers in SFO (2016)

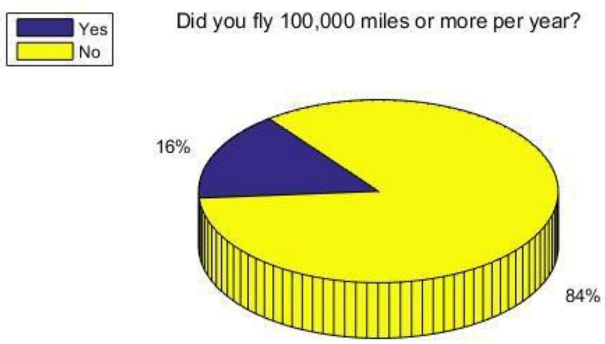


Figure 11: The flying miles distribution of passengers in SFO (2016)

But to further analyze the relationship between these four variables and the security check speed, we need to apply *the Theory of Grey Box System*. The theory of grey box system focus on researching the relationship of variables that are logically relevant but hard to figure out the exact mechanism. Some statistic method such as Regression Analysis, Variance Analysis and Principal Component Analysis may not apply to these situation, so the theory of grey box system introduces another highly recommended method called Correlation Analysis (Ju long, Deng,1999). [5]

To analyze the correlation degree, first we need to use the process of initialization transformation below:

$$f(x(k)) = \frac{x(k)}{x(1)} = y(k), \quad x(1) \neq 0 \tag{16}$$

After using the formula to eliminate the influence of magnitude, we need to determine a reference sequence

$$x_0 = \{x_0(k) \mid k = 1,2,\cdots,n\} = (x_0(1), x_0(2), \cdots, x_0(n)) \tag{17}$$

In the real practice, the reference sequence is the security check time of each passenger, which is the column V in the Excel file of SFO data. Then we use k to represent time point, assuming that there are m comparison sequence of variable value

$$x_i = \{x_i(k) \mid k = 1,2,\cdots,n\} = (x_i(1), x_i(2), \cdots, x_i(n)), \quad i = 1,2,\cdots,m \tag{18}$$

Then we call

$$\xi_i(k) = \frac{\min_s \min_t |x_0(t) - x_s(t)| + \rho \max_s \max_t |x_0(t) - x_s(t)|}{|x_0(k) - x_i(k)| + \rho \max_s \max_t |x_0(t) - x_s(t)|} \tag{19}$$

as the correlation degree between comparison sequence x_i and reference x_0 time point k.

The value of variable $\rho \in [0,1]$ here is arbitrary, using a larger value will lead to a clearer discrimination. After testing several times, we determine that $\rho = 0.8$. But $\xi_i(k)$ here varies according to the time point, too many values are hard to compare and analyze, so we introduce the variable

$$r_i = \frac{1}{n}\sum_{k=1}^{n} \xi_i(k) \tag{20}$$

As the correlation degree between comparison sequence x_i and reference x_0.

Using the math tool above, we calculate the correlation degree between security check speed and four variables:

$$r_{age} = 0.665671 \quad , \quad r_{income} = 0.747041 \quad , \quad r_{gender} = 0.75886 \quad , \quad r_{flying} = 0.689262$$

It is quite surprising that **gender and income have more impact on the security check speed than age and flying**. In further research, we find that female often have more items in bags that need to be checked than male, which may slow down the check speed. And the income level influence whether the passenger would choose to use pre-check lanes, which also lead to the difference of check speed. The age impact is mainly about the slow-down effect on behavior when people aging, and flying miles determine the degree passengers be familiar with the security check process.

6.2 Female Privileged Lanes

In the Sensitivity Analysis, we find that Gender has the most impact on security check time among the four variables. Inspired by the news of Capital International Airport that the separate female privileged lanes can increase security check efficiency by 25%, we decide to follow the example. The lane has no intention of any discrimination, we only want to improve the efficiency as well as offer convenience, for female need more time to have their personal items check, some of which may be quite embarrassing to be checked by a male staff. [6]

In the following analysis, we assume the whole security efficiency improve by15%, the same as above. And all the arrival and service data comes from the 2016 SFO Customer Survey Data.

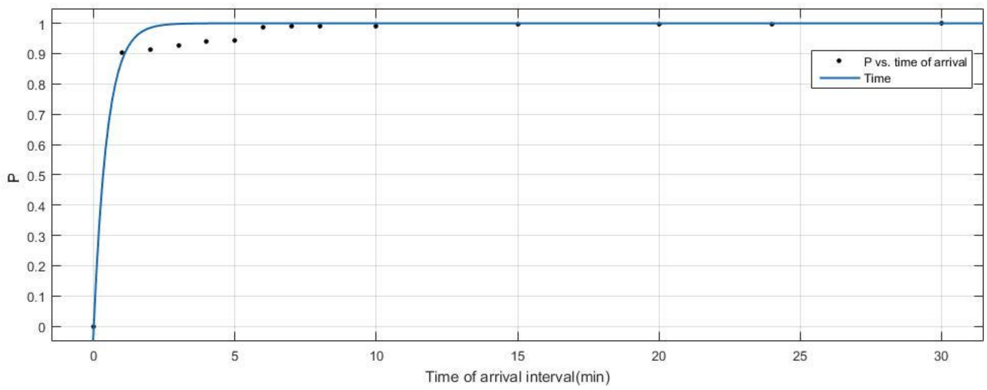


Figure 12: The fitting distribution of arrival intervals

The arrival parameter $\lambda = 2.107$

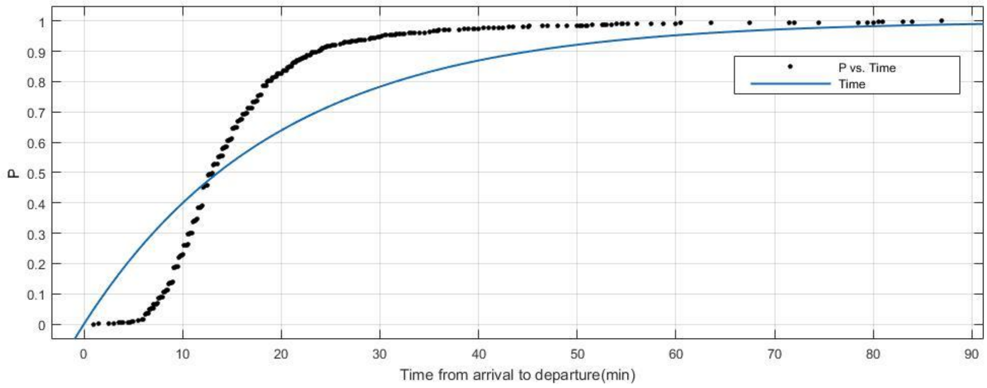


Figure 13: The fitting distribution of service time

The service parameter $\mu_{before} = 0.05097$

Because we assume that the service capacity of all lanes is bigger than the passenger arrival rate, the lane number $n \geq \frac{\lambda}{\mu_{before}} = 42$, hence, we assume that the lane number is 50.

After setting up the female privileged lanes, the service capacity increases by 25%,

so $\mu_{after} = 0.06371$

We can calculate the average length of the passenger line L_s, the average length of the queue L_q, the average stay time W_s and the average queue time W_q. From the value below, we can find that that the average waiting time can be largely shortened by 65%.

	Before	After
Value of variables	$L_s = 4.77$ $L_q = 3.95$ $W_s = 113.25$ $W_q = 93.63$	$L_s = 1.95$ $L_q = 1.29$ $W_s = 46.36$ $W_q = 30.66$

7 Proposal on policy and procedure/Conclusion

Based on our model, we sincerely propose the following suggestion to the security manager of each airport facing the problem of jam in the security check process. And our proposal focus on universality, so they can be globally applied. Meanwhile, they may not adapt to certain culture or country so well, so we hope that the security manager can combine these suggestions with the real situation.

- Centralized queue has a higher efficiency than the decentralized one. So we suggestion the airport utilize the one-queue process to decrease the waiting time for passengers, cost and site permitted.
- Offering personalized service to weak groups or passengers with special need will help reduce

not only their waiting time, but also the one of regular passengers.

- We suggest that the airport collect accurate statistics of every step of the security check and use our model to locate the exact bottleneck. Then properly adjust the number and proportion of different facilities to solve the problem of low usage rate of the facilities.
- We also suggest that the airport set up special facilities at the entrance to monitor the passenger flow, and adjust the number of lanes provide to passengers forthwith, to prevent retention of passengers in the airport and improve the reaction speed of the whole airport system.
- Airport should also focus on the administration of security check staffs, timely get to know the work state and efficiency of staffs, and promise the work quality of them.
- Put more efforts on getting the public familiar with the airport process, once the passengers are familiar with the steps of security check, the speed can be improved.
- Airport should focus on details as well, solving the linking problem of different process, and properly arrange the facility location can also reduce time for passengers to a large extent.

8 Strength and Future Improvement

8.1 Strength

- We transfer the multi-singular server model to multi-server model by properly using the queuing theory. The optimization has strong math support.
- We use specific variable W_q (the average length of passenger line)to measure the bottleneck, making an abstract concept quantitative.
- The fitting process of each step in model is very successful, good data resources lay stable foundation for our analysis.
- Correlation Degree performs very well on analyzing the impact variables of passengers' background brought to security check time.
- The modification of our model step by step show exactly the mind path of our research, and the final modification combines the improvement based on check system with the impact of passenger characteristics, we can call it a quite well-rounded model.
- The idea of female privileged lanes offer a brand new perspective, that we can improve the security check speed by offering more personal and passenger-oriented service.

8.2 Future improvement

Model of Current Process

- In the model, we assume that passenger arrivals come from an infinite or very large population, but the fact is apparently different. the arrivals have a limit.
- The actual passenger arrivals fluctuate with the flight times, so to increase accuracy, a better choice is to divide a day into several time periods and research the arrival process separately.
- We assume that the lanes provided by airport can always meet the demand of passengers, which means that the queue can be shortened to zero despite the low speed. But the reality is that the service capacity of lanes cannot meet the demand, when the passenger load is very large in the rush hours.

Model of Modification

- Until now, though we figure out a plan to improve the lane use rate, we haven't set up an exact measure criterion.
- We can consider adding cost variable into account and develop a model which can successfully balance the cost of updating equipment and the loss of passengers due to low security check efficiency.
- Security factor need to be included in the future, for now we only considered the number of facilities, but neglect the increase of work load on staffs. The shortage of human resource may lead to insecurity.

Model Based on Passenger Characteristics

- More variables about the personal character of passengers can be included. Considering more parameters will increase the stability of our model. But until now, we fail to access more statistics about the background of each passenger.
- We only finish the basic correlation analysis to know that some variables of passenger background have impact on security check speed, but whether the impact is positive or negative remains unknown. Also, the mechanism behind need to be explore.
- To set up female privileged lanes may arouse some social objection, for gender problem is very sensitive in the United States. Though we promise that women themselves remain the choice of whether using privileged lanes, the real effect is unknown.

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